

Optimization of Passively Damped Composite Structures

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ABSTRACT

A new technology, Stress Coupling Activated Damping (SCAD[©]), has been developed at Brigham Young University. Using stress coupling fiber reinforced composites with co-cured viscoelastic damping, damping may be increased from the undamped structures nominal 1% to more than 10% with some slight reductions in strength and stiffness. Through optimization, a designer may create highly damped, lightweight, and dynamically stiff components to solve many structural engineering problems. A computer model allows optimization for either stiffness, frequency or loss factor of a wide range of designs. One design resulted in a loss factor of 1.49%. When the same design was optimized for strength and stiffness with constraints on loss factor, the loss factor jumped to 5.11% with an increase in strength of 400% and stiffness of 1000%. An initial experiment has shown that this modelling method predicted resonant frequencies within 3%.

1. INTRODUCTION

There are two possible damping methods for controlling vibrations: passive damping and active damping. Viscoelastic materials used in passive damping methods dissipate less than ideal amounts of thermal energy. The need for a power source, force transducers, and feedback controls makes active damping methods difficult and expensive to implement.

Reinforced composites have an advantage over conventional materials in many applications because of their favorable strength to weight ratios, corrosion resistance, and unique stress coupling properties. Unfortunately, like isotropic materials such as steel and aluminum, composites by themselves do not exhibit significant damping. Viscoelastic materials, on the other hand, exhibit poor strength to weight ratios, but can provide impressive levels of damping when significant shear forces are generated in the material. At Brigham Young University, a new damping concept has been developed called Stress Coupling Activated Damping (SCAD[©]) which uses the desirable properties of both materials to create a lightweight, stiff, and highly damped structure. By optimizing this technology through a computer model, a maximum loss factor may be achieved without significant reductions of either strength or stiffness. This is achieved by placing constraints on the strength and stiffness requirements in the optimization program.

This paper presents the theory of viscoelastic fiber reinforced composites used in the construction of highly

damped structural components. Next, experimental results showing the reduction of vibrations using the SCAD[©] technology are presented. Finally, the optimization of a design for a composite tube is introduced.

2. RELATED RESEARCH

One of the more common passive damping technologies is called Constrained Layer Damping (CLD) which was developed and refined in the late 1950's and 1960's [1-4]. Several other researches have continued to develop this theory. CLD is achieved by bonding a thin, constrained layer of a metal sheet to an existing structure with a viscoelastic adhesive. Internal friction in the form of hysteresis losses in the viscoelastic material generates thermal energy, reducing the vibrational energy through the dissipation of heat. There are three major disadvantages of CLD: it adds weight and bulk, it may only be applied to the surface of the structure, and it is effective only for out-of-plane vibrations.

Barrett used the inherent shear coupling properties of composite materials to design a damped composite tubular component [5]. Constructing a plate with a layer of positive fiber angle orientation, a viscoelastic material layer, and a layer of negative fiber angle orientation will generate large shear strains when an axial load is applied. This lay-up sequence will cause shear coupling, which comes from the coupling of shear strains and normal loads.

Olcott developed a new damping concept called Stress Coupling Activated Damping (SCAD[©]) at Brigham Young University [6]. SCAD[©] uses the stress coupling effect of anisotropic materials, such as fiber reinforced composites, to distribute damping through the entire volume of embedded viscoelastic layers. The fiber orientation angle in each stiffness layer is alternated several times along the length of the component. Each time the fiber orientation angle is altered, a region of high shear is generated across the damping layer (see figure 1).

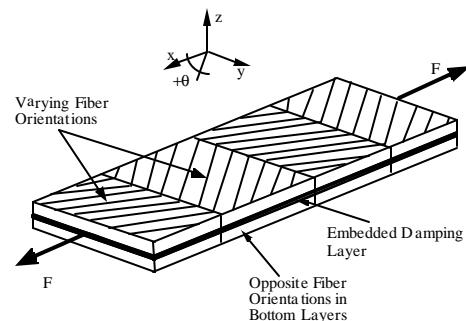


Figure 1. Olcott's Damping Concept.

3. ANALYTIC MODELING

An analytical model was developed to analyze these passive damping composite structures. The model, developed by Olcott, determines the loss factor and effective modulus of components.

Olcott's model, which determines the loss factor and the effective modulus of components and overall structures, uses the following assumptions for analysis [7]:

- 1) thin stiffness layers so that in-plane stresses and displacements did not vary through its thickness,
- 2) currently, the membrane may only be loaded axially and in transverse shear,
- 3) constant reactions and properties through the y-direction, therefore $du/dy = dv/dy = 0$,
- 4) the damping layer moduli are much lower than the stiffness layer moduli and hence normal loads applied to the stiffness layers are assumed negligible,
- 5) the only out-of-plane stress applied to the stiffness layers by the damping layers is the shear stresses τ_{xz} and τ_{yz} , which act through the neutral axis of the structure,
- 6) there is no slip at the interface between a stiffness layer and a damping layer, thus no delamination is assumed.

The general orthotropic stress-strain relationships applicable to composite layers in matrix notation are:

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{61} & \bar{Q}_{62} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{bmatrix}$$

The constants \bar{Q}_{11} through \bar{Q}_{66} are functions of the axial modulus, transverse modulus, poisson ratio, shear modulus, and the fiber orientation angle. Olcott then developed the governing equations for a single stiffness layer and combined them into a non dimensionalized matrix form :

$$\begin{bmatrix} \hat{K}_{11} & \hat{K}_{16} \\ \hat{K}_{16} & \hat{K}_{66} \end{bmatrix} \begin{bmatrix} \frac{\check{Z}\hat{u}}{\check{Z}x^2} \\ \frac{\check{Z}\hat{v}}{\check{Z}x^2} \end{bmatrix} + D_o \begin{bmatrix} 1 & 0 & -2 & 0 & 1 & 0 \\ 0 & 1 & 0 & -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{u}_{j-1} \\ \hat{v}_{j-1} \\ \hat{u}_j \\ \hat{v}_j \\ \hat{u}_{j+1} \\ \hat{v}_{j+1} \end{bmatrix} = 0 \quad (1)$$

where:

$$\hat{K}_{pq} = \frac{K_{pq}}{E_{11}} \quad \text{a function of the modulus } E_{11}, \text{ density and orientation of the stiffness layer}$$

$$\hat{u} = \frac{u}{L_s} \quad \text{axial direction displacements } u, \text{ divided by the segment length } L_s$$

$$\hat{v} = \frac{v}{L_s} \quad \text{transverse direction displacements } v, \text{ divided by the segment length}$$

$$\hat{x} = \frac{x}{L_s} \quad x \text{ is the axial direction divided by the segment length}$$

$$\frac{\check{Z}^2 u}{\check{Z}x^2} = \left(\frac{1}{L_s} \right) \frac{\check{Z}^2 \hat{u}}{\check{Z}x^2} \quad \text{by applying the partial derivative chain rule}$$

$$\frac{\check{Z}^2 v}{\check{Z}x^2} = \left(\frac{1}{L_s} \right) \frac{\check{Z}^2 \hat{v}}{\check{Z}x^2} \quad \text{by applying the partial derivative chain rule}$$

$$D_o = \frac{G_d L_s^2}{E_{11} t_s t_d} \quad G_d = \text{damping material moduli. } t_s = \text{thickness of the stiffness layer. } t_d = \text{thickness of the damping layer.}$$

Note that all properties are in complex form. The solution to the equation is of the form:

$$\begin{bmatrix} \hat{u}_1 \\ \hat{v}_1 \\ \dots \\ \dots \\ \hat{u}_n \\ \hat{v}_n \end{bmatrix} = \sum_{j=1}^n \begin{bmatrix} \hat{u}_1 \\ \hat{v}_1 \\ \dots \\ \dots \\ \hat{u}_n \\ \hat{v}_n \end{bmatrix} \left[\hat{c}_{+j} e^{(\lambda)_j \hat{x}} + \hat{c}_{-j} e^{(-\lambda)_j \hat{x}} \right] \quad (2)$$

The constants \hat{c}_{+j} and \hat{c}_{-j} are found by applying the appropriate load and displacement boundary conditions of the damped component being modeled. Particular analysis elements may be found which allow a small section of the whole component to be efficiently modeled; in return it will then yield the solution to the entire original component. For example, for free-free structures the boundary conditions are: at boundaries between each segment the displacements in both the x and y- directions must be equal, as must the normal and shear loads. Also, at either end of any component, assuming no body forces or acceleration, the axial and shear loads must be equal and opposite.

From the solutions to equation (2) the frequency and loss factor are calculated by the equations:

$$f = \frac{1}{2L_p} \sqrt{\frac{E_{eff}}{\rho}} \quad (3)$$

$$\eta = \frac{E'}{E''} \quad (4)$$

4. EXPERIMENTATION

Several passively damped composite tubes were made to test the analytic model. The composite tubes were tested in transverse vibration at Brigham Young University using Andruillie's composite tube test method [4]. The tubes were suspended at each end by soft rubber bands to approximate free-free boundary conditions. Transverse motion of the tube was monitored by attaching the accelerometer to one end, then striking the opposite end with a force hammer (see Figure 2). The force and acceleration signals were fed into the structural analyzer, which calculated the transfer function. The tube was struck 32 times, and the data averaged by the HP5423A analyzer. Damping values were calculated using the half-power method.

Table 1 shows the predicted versus the measured results for both the resonant frequency and the loss factors of a conventional composite tube and two of the several designs for damped composite tubes. t_d is per above. θ is the angle of the fibre orientation and L_p is the length of the part.

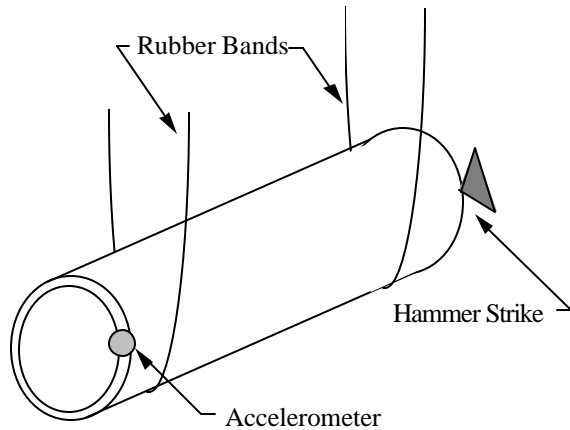


Figure 2. Data acquisition setup.

Two conclusions may be made from Table 1. First, the two tube designs using the SCAD© technology greatly improved the damping over the benchmark, or conventional composite tube design. The resonant frequencies were lowered and loss factor increased by as much as 200%, going from 2.36% to 5.56% damping. Secondly, the program can predict within 3% for resonant frequencies and 15% for loss factors. The predictions for the damped tubes are not as accurate as for the conventional composite tubes because manufacturing methods for the damped tubes are not as developed as they are for conventional tubes. Thus, manufacturing errors affected the loss factor measurements. Also, the material properties inputted for design calculations have a 10% error due to measurement methods. These reasons suggest why the loss factor is inaccurate in its prediction by up to 15%.

Table 1. Measured and predicted frequencies and loss factors for tubes.

Tube Specimens (Damping Material/ t_d / θ / L_s / L_p)	Measured		Predicted	
	(kHz)	(%)	(kHz)	(%)
NONE/N. A./26°/NONE/20.04"	6.75	2.36	6.56	2.02
ISD112/0.005"/26°/2.5"/20.93"	5.16	5.56	5.45	5.10
ISD112/0.005"/16°/2.5"/21.80"	6.41	2.77	6.58	2.73

5. OPTIMIZATION

A program has been developed which analyzes multiple angles (within a single stiffness layer) and variable damping layer properties using the analytic model. This program is combined with OptdesX, an X-Windows based optimization program developed by Design Synthesis, Inc.. Equations (1) thru (4) are programmed into OptdesX. During optimization, OptdesX changes values of the analysis variables (variables inputted into the design model) and passes the new variables into the design model. The design model, consisting of equations 1 and 2, calculates new analysis functions (damping, stiffness, and strength) and passes the functions back into OptdesX. This continues until design objectives and constraints are met. The design and analysis procedure applies equally to damped membranes and to damped cylinders with thin walls [8].

The following is an example problem of the optimization for the design of a composite tube. The axial loss (damping) will be maximized, while the strength and effective axial modulus (stiffness) are set not to decrease below a specified limit. In this manner, one may obtain the most damping while meeting strength and stiffness requirements of the part. The initial analysis variables and resulting analysis functions are shown in Table 2. The analysis functions, except axial loss and frequency, have been normalized to E_1 .

Table 2. Initial analysis variables and resulting analysis functions from OptdesX.

Analysis Variables		Analysis Functions	
<i>Do</i>	0.319	<i>Axial Loss</i>	1.49
<i>Damp Frac</i>	0.75	<i>Eff. Axial Mod</i>	0.177
<i>Axial Stress</i>	1.0	<i>Shear Loss</i>	0.00
<i>Axial Mod</i>	1.76E7	<i>Strength</i>	0.033
<i>Trans Mod</i>	1.25E7	<i>Frequency</i>	3195
<i>Shear Mod</i>	8.5E5	<i>Stress</i>	0.503
<i>Poisson</i>	0.3	<i>Shear Stress</i>	-0.499
<i>Long Damp</i>	0.005		
<i>Length</i>	20.0		
<i>Density</i>	0.074		
<i>Angle</i>	45		

The optimization problem is defined by specifying design variables, design functions, objectives and constraints. First, analysis variables are mapped as design variables. In this case, the analysis variable *Angle* was chosen to vary. Other variables or multiple variables can be chosen as well. Next, the design variable is assigned maximum and minimum values. In this case, *Angle* was assigned a minimum angle of 10° and a maximum angle of 50°. The third step is to map analysis functions as design functions. The analysis

functions used as design functions for this example were *axial loss factor*, *effective axial modulus* (stiffness) and *strength*.

The three design functions need objectives and constraints. Objectives are signified as either minimizing, maximizing or not as an objective. For example, *Axial loss* was assigned a minimum value constraint of 5 but not an objective. The *strength* and *effective axial modulus* values were assigned a maximizing objective and a minimum constraint. Both *effective axial modulus* and *strength* were given minimum constraints corresponding to the initial analysis at 45°. These two design functions will never drop below their initial values because they are being “maximized.”

Finally, OptdesX changed the analysis variable *Angle* to meet the objectives and constraints described above. Table 3 below shows the optimized values for the analysis variables and analysis functions [5].

Table 3. Optimized analysis variables and analysis functions from OptdesX.

Analysis Variables		Analysis Functions	
<i>Do</i>	0.319	<i>Axial Loss</i>	5.11
<i>Damp Frac</i>	0.75	<i>Eff. Axial Mod</i>	0.73
<i>Axial Stress</i>	1.0	<i>Shear Loss</i>	0.00
<i>Axial Mod</i>	1.76E7	<i>Strength</i>	0.33
<i>Trans Mod</i>	1.25E7	<i>Frequency</i>	6475
<i>Shear Mod</i>	8.5E5	<i>Stress</i>	0.97
<i>Poisson</i>	0.3	<i>Shear Stress</i>	-0.17
<i>Long Damp</i>	0.005		
<i>Length</i>	20.0		
<i>Density</i>	0.074		
<i>Angle</i>	12		

Interestingly, the 33° angle change caused the axial loss to increase from 1.5% to 5.1% while almost quadrupling the effective axial modulus (0.18 to 0.73) and increasing the strength a hundred fold (0.03 to 0.33).

These results show that a designer may optimize for either strength, stiffness or damping of a composite structure within the limits of the constraints. This designing and optimization program also allows for the customization of frequency response for special applications. For example, if desired, a designer may create a composite structure which resonates at a frequency below a specified damaging resonant frequency or simply lower the resonant frequency to reduce vibrations while still maintaining specified strength and stiffness requirements.

6. CONCLUSIONS & RECOMMENDATIONS

Initial experiments of optimally designed composite tubes using the SCAD© technology have shown significant results. Optimization provided an increase in damping, strength and stiffness over preliminary designs. The models were also able to predict within 3% for resonant frequencies and 20% for loss factors. Models were thus proven adequate for designer estimates and can provide predictive analysis for designers as well as optimization of the design.

SCAD© technology can be applied to a wide range of designs. The geometry of the structure is not limited to just thin plates, but rather to several different geometries, including: tubes, thin plates, Ibeams, and other various geometries. This allows SCAD© technology to be applied to several design problems, including those in the metrology, medical, aerospace, automotive, machine tool, and other industries.

Research will continue to focus on improving the performance, manufacturability, and extending the uses of SCAD© related technology and concepts. The following areas have also been identified:

- refine and verify analytic models,
- develop the optimization of multiple damping layers,
- and improve bending equations for the prediction models.

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